EXAMINATIONS OF THE HONG KONG STATISTICAL SOCIETY

HIGHER CERTIFICATE IN STATISTICS, 2014

MODULE 7 : Time series and index numbers

Time allowed: One and a half hours

Candidates should answer **THREE** *questions.*

Each question carries 20 marks.
The number of marks allotted for each part-question is shown in brackets.

Graph paper and Official tables are provided.

Candidates may use calculators in accordance with the regulations published in the Society's "Guide to Examinations" (document Ex1).

The notation log *denotes logarithm to base* ***e***.
Logarithms to any other base are explicitly identified, e.g. $\log_{10}$.

Note also that $\binom{n}{r}$ *is the same as* ${}^{n}C_{r}$.

This examination paper consists of 8 printed pages.
This front cover is page 1.
Question 1 starts on page 2.

There are 4 questions altogether in the paper.

1. (i) State the three main components into which a time series is usually decomposed for analysis. Explain what each represents.

(6)

(ii) Give simple equations to show how a time series may be represented by

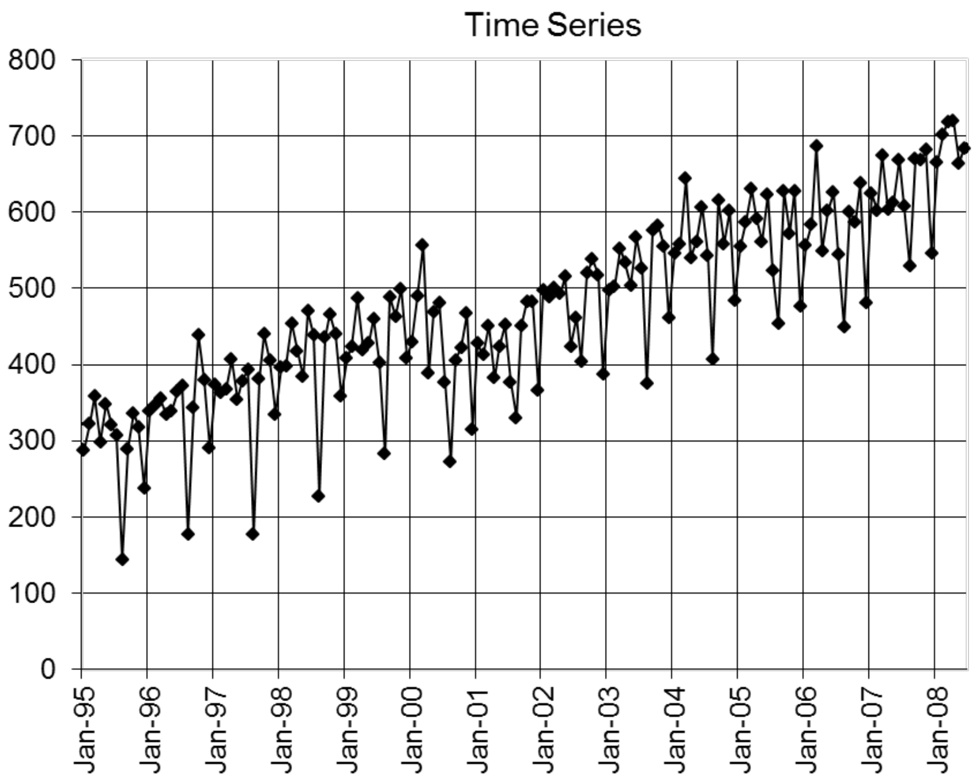
(a) an additive decomposition,

(b) a multiplicative decomposition.

(2)

(iii) The graph shows a monthly time series. State, with reasons, whether you think that it would be better represented by an additive or multiplicative decomposition. Describe and explain circumstances in which a multiplicative decomposition would be impossible.

(3)



(iv) The 'Easter effect' in UK time series relates to the moving nature of Easter Sunday (which falls between 22 March and 25 April, and generally moves between March and April at least every other year) and hence also to the moving nature of the public holidays granted two days before and one day after Easter Sunday. Explain why Easter is more likely to cause complications than fixed date holidays such as Christmas in the analysis of either monthly or quarterly time series.

(3)

Question continued on the next page

(v) In the regression analysis of a time series, the Easter effect had a parameter estimate of –29.3017 with standard error 5.36264. Discuss whether an adjustment for Easter should be made.

(2)

(vi) The following table shows the March and April values for two seasonally adjusted versions, *Y1* and *Y2*, of the same time series. This series tends to have smaller values in months that include public holidays than in months that do not. In the case of *Y1* an adjustment was made for the effects of Easter; for *Y2* no Easter adjustment was made. Suggest, with reasons, in which years Easter was early and in which years it was late.

(4)

	March				*April*		
	Y1	*Y2*	*Y1 – Y2*		*Y1*	*Y2*	*Y1 – Y2*
1995	305.72	307.94	–2.22	1995	293.83	292.66	1.17
1996	310.95	303.48	7.48	1996	321.45	328.64	–7.18
1997	349.24	312.81	36.43	1997	366.88	402.43	–35.54
1998	393.64	396.02	–2.38	1998	421.60	416.78	4.82
1999	449.70	427.14	22.55	1999	404.73	422.06	–17.34
2000	492.95	494.45	–1.50	2000	404.82	391.94	12.88
2001	385.98	387.61	–1.64	2001	402.69	385.41	17.28
2002	472.08	436.35	35.73	2002	475.11	492.47	–17.35
2003	478.48	486.36	–7.88	2003	554.65	529.49	25.16
2004	563.40	576.36	–12.96	2004	559.11	531.58	27.53
2005	581.56	559.90	21.66	2005	567.81	582.87	–15.07
2006	595.13	616.26	–21.13	2006	560.29	540.09	20.19
2007	586.63	605.12	–18.49	2007	606.32	594.65	11.68
2008	666.98	650.94	16.04	2008	684.59	709.39	–24.79

2. The data below represent shop sales of a mobile phone over a three-year period. In order to decide whether to continue selling this model, the shop owners need to understand the underlying movement in this time series. To help them, you need to choose an appropriate smoothing method.

2010 Q1	13
2010 Q2	15
2010 Q3	18
2010 Q4	19
2011 Q1	21
2011 Q2	22
2011 Q3	23
2011 Q4	25
2012 Q1	23
2012 Q2	19
2012 Q3	15
2012 Q4	12

(i) A 2 × 4 moving average is defined as a simple 2-point moving average of a simple 4-point moving average. Calculate the weights associated with such a moving average and apply it to the time series in the table above. Give exact answers.

(7)

(ii) Write down the equation used in simple exponential smoothing, which relates the one-step-ahead forecast of a time series at time t to the one-step-ahead forecast at time $t-1$ and the value of the series at time t. Taking the smoothing parameter in this equation as 0.5, apply the technique to the time series in the table above. Give your answers to three decimal places.

(7)

(iii) The series of one-step-ahead forecasts produced by simple exponential smoothing can be used as an estimate of the trend in a series. Describe an advantage and a disadvantage of using exponential smoothing as implemented in part (ii) to estimate trend, compared with using the moving average in part (i), for the time series in the table above.

(4)

(iv) Discuss whether the choice of 0.5 for the smoothing parameter in part (ii) is a sensible one.

(2)

3. Suppose a country has only one mobile telephone network. The table below gives some data on the sales of units (a unit being either a single text message or a one-minute telephone call) to domestic customers and to roaming customers (foreign visitors using the network). Use these data in your answers to the following questions.

(i) Calculate the Laspeyres, Paasche and Fisher price indices for 2012 to two decimal places, using 2011 as the base period.

(10)

(ii) State what appears to be surprising about these indices, and give a possible explanation.

(2)

(iii) Given the indices calculated above, state what is the simplest way of calculating the Laspeyres, Paasche and Fisher volume indices for 2012, using 2011 as the base period.

(3)

(iv) Calculate the Laspeyres, Paasche and Fisher volume indices for 2012, using 2011 as the base period in the way you have just described.

(5)

Commodity	*Quantity of units sold in 2011 (millions)*	*Value of units sold in 2011 (£million)*	*Quantity of units sold in 2012 (millions)*	*Value of units sold in 2012 (£million)*
Domestic units	100.6	10.00	120.0	11.04
Roaming units	1.0	1.00	1.9	1.87

4. (i) Prove that the Laspeyres price index with current period 0 and base period 1 is the reciprocal of the corresponding Paasche price index with current period 1 and base period 0.

(3)

(ii) Prove that the Paasche price index with current period 0 and base period 1 is the reciprocal of the corresponding Laspeyres price index with current period 1 and base period 0.

(3)

(iii) Using the results of parts (i) and (ii), prove that the Fisher price index with current period 0 and base period 1 is the reciprocal of the Fisher price index with current period 1 and base period 0.

(4)

(iv) The formula for the Törnqvist price index is

$$P_T(0,1) = \prod_i \left(\frac{p_{1i}}{p_{0i}} \right)^{\frac{1}{2}(w_{0i}+w_{1i})}$$

where 0 is the base period,
1 is the current period,
p denotes price,
w denotes expenditure share and
i identifies an individual item.

Prove that the Törnqvist price index with current period 0 and base period 1 is the reciprocal of the Törnqvist price index with current period 1 and base period 0.

(6)

(v) Starting from the formulae for the Laspeyres price index and the Paasche price index in terms of prices and quantities, show how they can be re-expressed in terms of price relatives and expenditure.

(4)

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