EXAMINATIONS OF THE HONG KONG STATISTICAL SOCIETY

HIGHER CERTIFICATE IN STATISTICS, 2011

MODULE 2 : Probability models

Time allowed: One and a half hours

Candidates should answer **THREE** *questions.*

Each question carries 20 marks.
The number of marks allotted for each part-question is shown in brackets.

Graph paper and Official tables are provided.

Candidates may use calculators in accordance with the regulations published in the Society's "Guide to Examinations" (document Ex1).

The notation log *denotes logarithm to base* ***e***.
Logarithms to any other base are explicitly identified, e.g. $\log_{10}$.

Note also that $\binom{n}{r}$ *is the same as* ${}^nC_r$.

This examination paper consists of 4 printed pages, **each printed on one side only**.
This front cover is page 1.
Question 1 starts on page 2.

There are 4 questions altogether in the paper.

1. Clay pots made at a pottery are subject to three types of defect. It is found that 10% of pots show brittle fracture (B), 4% have cracked glazing (C) and 10% are discoloured (D).

(i) Assuming that all three types of defect occur independently, what is the probability that a randomly chosen pot has no defects?

(4)

(ii) Experience has shown that the three types of defect do not all occur independently. 20% of pots with brittle fracture also have cracked glazing, but both of these defects occur independently of discoloration.

(a) Find the probability that a pot has both brittle fracture and cracked glazing, and hence find the probability that a pot has either or both of these defects.

(5)

(b) What is now the probability that a randomly chosen pot has no defects?

(5)

(c) Suppose that a pot does not have cracked glazing and is not discoloured. Find the probability that it has brittle fracture.

(6)

2. (i) A sequence of three plus (+) signs and two minus (–) signs is written in a straight line. Write down all 10 possible sequences (different orders) of these signs.

(5)

(ii) A "run" of one sign is a series of one or more consecutive instances of this sign, terminated at each end either by the opposite sign or by the beginning or end of the sequence. Assume that sequences of these signs are generated randomly, so that each possible sequence has probability 0.1 of occurring, and let the random variable X denote the number of runs in a randomly chosen sequence. For example, if the sequence − + − + + is observed then $X = 4$: one run of one −, one run of one +, one run of one −, one run of ++.

(a) Write down the values taken by X for each of the 10 sequences you have listed in part (i), and hence obtain the probability distribution of X.

(5)

(b) Find $E(X)$ and $\mathrm{Var}(X)$.

(10)

Turn over

3. In a certain large population, the heights X in cm of adult males have a Normal distribution with mean 170 and variance 100.

(i) (a) Find the probability that a randomly chosen man is within 5 cm of the population average height.

(2)

(b) Find the probability that a randomly chosen man is more than 185 cm tall.

(2)

(c) Given that a randomly chosen man is more than 185 cm tall, find the probability that he is more than 189.6 cm tall.

(2)

(ii) Four men are chosen at random from this population.

(a) Find the probability that all four are within 5 cm of the population average height.

(2)

(b) Find the probability that exactly two of the four are within 5 cm of the population average height.

(3)

(c) Find the probability that the average height of the sample of four is within 5 cm of the population average height.

(3)

(iii) You are given that $P(X > 189.6) = 0.025$. Given a random sample of 100 men from the population, let Y denote the number of men in the sample who are over 189.6 cm tall. State

(a) the exact distribution of Y,

(b) a suitable approximation to the exact distribution of Y.

Use this approximate distribution to find the value of $P(Y > 3)$.

(6)

4. The random variable X has the geometric distribution with probability mass function (pmf)

$$p_X(x) = q^x p, \quad x = 0, 1, 2, \ldots, \quad 0 < p < 1,$$

where $q = 1 - p$.

(i) Find $P(X \geq x)$ and show that $P(X \leq 3) = 1 - q^4$.

(3)

(ii) Explain why

$$P(X \text{ is odd } (= 1, 3, 5, 7, \ldots)) = q \times P(X \text{ is even } (= 0, 2, 4, 6, \ldots))$$

and hence or otherwise show that

$$P(X \text{ is odd}) = \frac{q}{1+q}.$$

(3)

(iii) Find $P(X \text{ is odd} \mid X \leq 3)$ as a function of q, and, given that

$$P(X \text{ is odd} \mid X \leq 3) = \tfrac{1}{3},$$

deduce the value of q.

(6)

(iv) The random variable Y is independent of X and has pmf

$$p_Y(y) = qp^y, \quad y = 0, 1, 2, \ldots.$$

Write down $P(X = k \text{ and } Y = k)$ for an arbitrary non-negative integer k and hence show that

$$P(X = Y) = \frac{p(1-p)}{1 - p(1-p)}.$$

Using calculus, find the value of p that maximises this expression and hence deduce that the maximum possible value of $P(X = Y)$ is $\tfrac{1}{3}$.

[Note. You are not required to show that any turning point that you locate is a maximum.]

(8)